

Scalar form factor of the pion in the Kroll-Lee-Zumino field theory

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The renormalizable Kroll-Lee-Zumino field theory of pions and a neutral rho-meson is used to determine the scalar form factor of the pion in the space-like region at next-to-leading order. Perturbative calculations in this framework are parameter free, as the masses and the rho-pion-pion coupling are known from experiment. Results compare favorably with lattice QCD calculations.

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The scalar form factor of the pion [1], and particularly its quadratic radius, plays an important role in chiral perturbation theory (CHPT) [2]. This form factor is defined as the pion matrix element of the QCD scalar current $J_S = m_u \bar{u}u + m_d \bar{d}d$, i.e.

$$F_S(q^2) = \langle \pi(p_2) | J_S | \pi(p_1) \rangle, \quad (1)$$

where $q^2 = (p_2 - p_1)^2$. The associated quadratic scalar radius is given by

$$F_S(q^2) = F_S(0) \left[1 + \frac{1}{6} \langle r_\pi^2 \rangle_S q^2 + \dots \right], \quad (2)$$

where $F_S(0)$ is the pion sigma term

$$F_S(0) \equiv \sigma_\pi = m_q \frac{\partial M_\pi^2}{\partial m_q}. \quad (3)$$

The scalar radius fixes $\bar{\ell}_4$, one of the low energy constants of CHPT, through the relation

$$\langle r_\pi^2 \rangle_S = \frac{3}{8\pi^2 F_\pi^2} \left[\bar{\ell}_4 - \frac{13}{12} + O(M_\pi^2) \right], \quad (4)$$

where $F_\pi = 91.9 \pm 0.1$ MeV is the physical pion decay constant [3]. The low energy constant $\bar{\ell}_4$, in turn, determines the leading contribution in the chiral expansion of the pion decay constant, i.e.

$$\frac{F_\pi}{F} = 1 + \left(\frac{M_\pi}{4\pi F_\pi} \right)^2 \bar{\ell}_4 + O(M_\pi^4), \quad (5)$$

where F is the pion decay constant in the chiral limit. This scalar form factor is not accessible experimentally, but it has been determined from lattice QCD (LQCD) [4]-[6], or hadronic models [7].

Theoretically, the ideal tool to study this form factor, independently from LQCD, is the Kroll-Lee-Zumino Abelian renormalizable field theory of pions and a neutral ρ -meson [8]. This provides the appropriate field theory platform for the phenomenological Vector Meson Dominance (VMD) model [9], allowing for a systematic calculation of higher order quantum corrections [10]-[11]. Due to the renormalizability of the theory, predictions are parameter free, as the strong $\rho\pi\pi$ coupling, $g_{\rho\pi\pi}$, is known from experiment. In spite of this coupling being a strong interaction quantity, perturbative calculations in the \overline{MS} scheme make sense because the effective expansion parameter turns out to be $(g_{\rho\pi\pi}/4\pi)^2 \simeq 0.2$. The KLZ theory has been used to compute the next-to-leading order (NLO) correction to the tree level (VMD) electromagnetic form factor of the pion in the space-like region with very good results [10]. In fact, it agrees with data up to $q^2 \simeq -10 \text{ GeV}^2$ with a chi-squared per degree of freedom $\chi_F^2 = 1.1$, as opposed to VMD which gives $\chi_F^2 = 5.0$. In addition, the mean-squared radius at NLO is $\langle r_\pi^2 \rangle = 0.46 \text{ fm}^2$, compared with the experimental result [3] $\langle r_\pi^2 \rangle = 0.45 \pm 0.01 \text{ fm}^2$, and the VMD value $\langle r_\pi^2 \rangle = 0.39 \text{ fm}^2$.

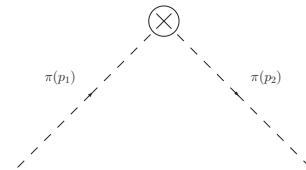


FIG. 1: Leading order (LO) contribution to the scalar form factor of the pion. The cross indicates the coupling of the scalar current to two pions.

In this note we compute in this framework the scalar form

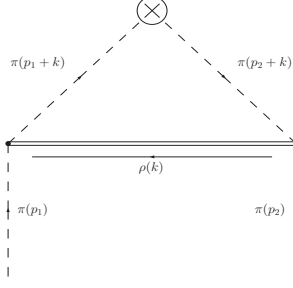


FIG. 2: Next-to-leading order (NLO) contribution to the scalar form factor.

factor of the pion at NLO in the space-like region, and compare with current results from LQCD. The KLZ Lagrangian is given by

$$\begin{aligned} \mathcal{L}_{KLZ} = & \partial_\mu \phi \partial^\mu \phi^* - M_\pi^2 \phi \phi^* - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} M_\rho^2 \rho_\mu \rho^\mu \\ & + g_{\rho\pi\pi} \rho_\mu J_\pi^\mu + g_{\rho\pi\pi}^2 \rho_\mu \rho^\mu \phi \phi^* , \end{aligned} \quad (6)$$

where ρ_μ is a vector field describing the ρ^0 meson ($\partial_\mu \rho^\mu = 0$), ϕ is a complex pseudo-scalar field describing the π^\pm mesons, $\rho_{\mu\nu}$ is the usual field strength tensor: $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$, and J_π^μ is the π^\pm current: $J_\pi^\mu = i\phi^* \overleftrightarrow{\partial}_\mu \phi$. In spite of the explicit presence of the ρ^0 mass term in the Lagrangian, the theory is renormalizable because the neutral vector meson is coupled to a conserved current [8]. Figures 1 and 2 show, respectively, the LO and the NLO diagrams, where the cross indicates the coupling of the current to the two pions. Notice that while the Lagrangian, Eq.(6), contains a $\rho\rho\pi\pi$ quartic coupling, this term only contributes in this application at NNLO and beyond.

Using the Feynman propagator for the ρ -meson, and in d dimensions, the unrenormalized vertex function in Fig.2 in dimensional regularization is given by

$$\begin{aligned} \tilde{G}(q^2) = & -2 \frac{g_{\rho\pi\pi}^2}{(4\pi)^2} (\mu^2)^{(2-\frac{d}{2})} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \\ & \times \left\{ \frac{2}{\varepsilon} - \ln \left(\frac{\Delta(q^2)}{\mu^2} \right) - \frac{1}{2} - \gamma + \ln(4\pi) \right. \\ & + \frac{1}{2\Delta(q^2)} [M_\pi^2(x_1 + x_2 - 2)^2 \\ & \left. - q^2(x_1 x_2 - x_1 - x_2 + 2)] + O(\varepsilon) \right\} , \end{aligned} \quad (7)$$

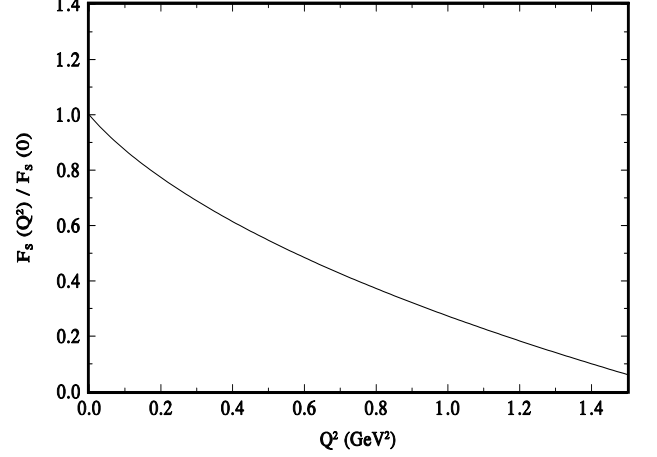


FIG. 3: The normalized scalar form factor, Eq.(8), to NLO in the space-like region.

where $\Delta(q^2)$ is defined as

$$\Delta(q^2) = M_\pi^2(x_1 + x_2)^2 + M_\rho^2(1 - x_1 - x_2) - x_1 x_2 q^2 . \quad (8)$$

In the \overline{MS} scheme, and renormalizing the vertex function at the point $q^2 = 0$, the NLO contribution in Fig. 2 is [11]

$$\begin{aligned} G(q^2) - G(0) = & -2 \frac{g_{\rho\pi\pi}^2}{(4\pi)^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \\ & \times \left\{ \ln \left(\frac{\Delta(q^2)}{\Delta(0)} \right) + \frac{1}{2} \left[M_\pi^2(x_1 + x_2 - 2)^2 \left(\frac{1}{\Delta(q^2)} \right. \right. \right. \\ & \left. \left. - \frac{1}{\Delta(0)} \right) - \frac{q^2}{\Delta(q^2)}(x_1 x_2 - x_1 - x_2 + 2) \right] \right\} , \end{aligned} \quad (9)$$

with

$$F_S(q^2) = F_S(0) [1 + G(q^2) - G(0)] . \quad (10)$$

For details on the renormalization procedure for the fields, masses and coupling see [10]. The result of a numerical evaluation of Eq.(9), using $g_{\rho\pi\pi}^2 = 36.0 \pm 0.2$ from the measured width of the ρ -meson [3], is shown in Fig.3. Regarding the scalar radius, defined in Eq.(2), we confirm the NLO result obtained in [11]

$$\langle r_\pi^2 \rangle_S = 0.4 \text{ fm}^2 , \quad (11)$$

with a negligible error due to the strong coupling.

This value is smaller than typical values in the literature [4]-[7]. However, it must be kept in mind that the NLO result is expected to be a lower bound, i.e. with $[G(q^2) - G(0)] < 0$ the NNLO would reduce $F_S(q^2)$, thus increasing the radius. A rough order of magnitude estimate of the size of the NNLO contribution suggests a correction of some 20% to the NLO term (the NNLO calculation is quite formidable and beyond the scope of this note). This is obtained by estimating a typical two-loop diagram, e.g. the ρ -meson propagator at NNLO and comparing it with the NLO result. The Feynman integrals in the variables x_i at NLO and NNLO are of order $\mathcal{O}(1)$ in the q^2 range explored here. We find the total contribution from this diagram to be over 20% of the NLO, thus increasing the radius to $\langle r_\pi^2 \rangle_S \simeq 0.5 \text{ fm}^2$.

A comparison of the KLZ form factor itself at low $|q^2| <$

0.5 GeV^2 with LQCD results read from figures in [4] and [6] shows good agreement. It should be mentioned, though, that LQCD results from [4] are for light-quark masses in the range from $m_s/6$ to $m_s/2$, while those from [6] are for $m_\pi = 325 \text{ MeV}$. These LQCD determinations find values for the scalar radius higher than in this analysis, Eq.(11), i.e. $\langle r_\pi^2 \rangle_S = 0.6 \pm 0.1 \text{ fm}^2$ from [4], and $\langle r_\pi^2 \rangle_S = 0.637 \pm 0.023 \text{ fm}^2$ from [6]. These results for the radius are determined from e.g. chiral extrapolations to the physical pion mass. Our results for the form factor are also in agreement within less than 10% with a CHPT calculation [12] in the range $-q^2 = 0 - 0.2 \text{ GeV}^2$.

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